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Jerome Bracken

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by

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U. S. DEPARTMENT OF HEALTH, EDUCATION, AND WELFARE
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DIVISION OF RADIOLOGICAL HEALTH
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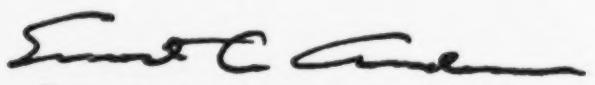
FOREWORD

A primary responsibility of the Division of Radiological Health is to make available information that will provide guidelines for procedures to minimize exposure of the population to radiation. It has become clear that under various circumstances higher exposures may occur as a result of the presence of radioactive nuclides in certain components of the diet. Since such exposures might call for measures to reduce them, it becomes useful to examine, among various possible measures, the feasibility of providing alternative foods that will minimize dietary exposure to radiation and at the same time maintain good nutrition.

A program of this nature has two distinct aspects. The first would be related to determining the kind or extent of damage that the ingestion of a given radionuclide may have on a biological system. The second is the development of a method by means of which a number of alternative diets may be designed to provide satisfactory nutritional value with minimal weighted radionuclide intake.

An evaluation of the biological effects of a given radionuclide could lead to a system of weights to be used in this problem, which requires the application of a numerical factor proportional to the biological effect of the radionuclide. Considerable attention is being devoted to this portion of the problem at many levels of scientific investigation. The study presented here is directed to the second aspect of the problem, and was undertaken by Dr. Jerome Bracken when he was with the National Bureau of Standards and serving as a consultant to the Division of Radiological Health on computation and automatic data processing systems.

In developing an objective means of selecting a diet that is adequate nutritionally and minimal in weighted radionuclide concentrations, practical considerations dictate that a choice of more than one diet be offered. The approach to the problem as described in this study is flexible in its ability to meet the changing dietary requirements that might be imposed. It should be made clear that the study offers no more than a mathematical model for attaining stated objectives which represent only a part of the solution, the balance of which will be dependent upon the development of a useful weighting scheme for radionuclides according to their biological effects.



Ernest C. Anderson, Chief
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ABSTRACT

Some motivating and background material underlying the formulation of the models is presented. Several mathematical programming models for selecting diets to minimize radionuclide intake are formulated, of which the most attractive seems to be for minimizing linearly weighted radionuclide intake. An example is given of the application of one of the models using published data of radionuclide content of foods.

MATHEMATICAL PROGRAMMING MODELS FOR SELECTION OF DIETS TO MINIMIZE WEIGHTED RADIONUCLIDE INTAKE[1]

by

Jerome Bracken[2]

Data Processing Systems Division, National Bureau of Standards

Work on the application of mathematical programming to the radiological health problems considered in this paper has been motivated by the interest of the U. S. Public Health Service in radioactivity intake from the diet. The Division of Radiological Health of the Public Health Service is concerned with measuring radiation in various environmental media (e.g., air, water, food), establishing information systems to process the measurements, interpreting the measured data, and developing public health guidelines.

The models presented in this paper provide a means for choosing diets which (a) contain nutrients in amounts equal to or greater than the recommended dietary allowances, and (b) contain minimum weighted radionuclide levels. Data which could be used in the models are presently being collected and processed; thus the models would require no great additional data processing efforts. The models could be used as an analytical framework for extending presently available information for monitoring, evaluative and other similar purposes.

An example is given in the paper which uses measurements of radioactivity in foods taken from published data (11). The example illustrates one of the models and one form of solution; it also serves to point out the necessary data organization, which could be a consideration in setting up processing procedures for these particular types of data.

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[1]The work described in this paper was performed under the sponsorship of the Division of Radiological Health, U. S. Public Health Service.

[2]Present address: Logistics Research Project, The George Washington University, Washington 6, D.C.

The models are of the same general class as the well known minimum-cost diet problem first published in 1945 by Stigler (10). Stigler's diet problem was one of the first problems of significantly large size to be solved by the simplex method, having 9 inequalities and 77 variables (9 nutrients and 77 foods). Stigler's model was posed for diets of human beings; Waugh (12) computed a minimum-cost dairy feed diet using a smaller model. An extensive discussion of the diet problem including various types of interpretations may be found in (4).

The basic diet problem may be extended in many ways, such as (a) to force selected foods into the solution in at least a minimum amount, (b) to put upper limits on the amount of a food which may be included, (c) to maximize nutrient content of a diet subject to cost constraints, (d) to formulate the criterion function in terms of several variables (e.g., cost and taste preferences), or (e) to search for alternate optimal solutions or convex combinations of these solutions. In this paper, we are concerned primarily with two extensions in model formulation which are believed to be immediately germane to the problem being studied:

1. A criterion function of minimizing intake of one radionuclide is given; then the criterion function is extended to minimizing weighted intake of several radionuclides.
2. Use is made of alternative diets which satisfy the constraints, and of alternate optimal diets in the forms of solution which are suggested in this paper.

MODELS

We are given the nutrient content of a number of different foods, and the minimum daily requirement for each nutrient. We define

a_{ij} = the concentration of the ith nutrient in the jth food,

b_i = the minimum daily requirement of the ith nutrient,

x_j = the daily amount of the jth food to be included in the diet.

If we consider n foods and m nutrients, and let

p_j = the price per unit of the jth food,

then the linear programming (the original diet) problem is to minimize

$$\sum_{j=1}^{n} p_j x_j$$

subject to

$$x_j \geq 0 , \qquad j = 1 , 2 , \ldots , n$$

and

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i , \qquad i = 1 , 2 , \ldots , m .$$

Minimizing Intake of One Radionuclide

Let us assume that we are constrained by the nutrient requirements defined above. It is possible by radiochemical techniques or by gamma spectrometry to measure the concentration of a particular radionuclide in each of the n foods being considered. Letting

c_j = the concentration of the particular radionuclide being considered in the jth food,

we may state the linear programming problem of minimizing the dietary intake of the one radionuclide as to minimize

$$\sum_{j=1}^{n} c_j x_j$$

subject to

$$x_j \geq 0 , \qquad j = 1 , 2 , \ldots , n$$

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and

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i , \qquad i = 1 , 2 , \ldots , m .$$

If we had available, for example, measurements of the strontium-90 concentration in each of n foods, we could find by solving the linear programming problem a diet which minimizes the intake of strontium-90 while satisfying the nutrient requirements. We could find other relevant diets and their strontium-90 content by methods to be discussed.

Minimizing Linearly Weighted Intake of Several Radionuclides

Again, let us assume the constraining nutrient requirements previously defined. Instead of minimizing the intake of one radionuclide, however, let us consider several radionuclides. It would be possible to measure the concentration of each of r radionuclides in the n foods. Let us define

c_{kj} = the concentration of the kth radionuclide in the jth food

and the intake

d_k = the amount of intake of the kth radionuclide.

For a given diet $x_j (j = 1, 2, \ldots, n)$, the intake of the kth radionuclide would be

$$d_k = \sum_{j=1}^{n} c_{kj} x_j , \qquad k = 1 , 2 , \ldots , r .$$

Consider a two-stage problem which involves the nutrient requirements and the dietary radionuclide intake. If we choose a diet to satisfy

$$x_j \geq 0 , \qquad j = 1 , 2 , \ldots , n$$

and

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \ , \qquad i = 1 , 2 , \ldots , m \ ,$$

this diet will contain radionuclide levels

$$d_k = \sum_{j=1}^{n} c_{kj} x_j \ , \qquad k = 1 , 2 , \ldots , r \ .$$

Some radionuclides are more damaging than others. It would be appropriate, therefore, to weight the levels of the radionuclides. Let us define the weight

w_k = the weight assigned to the kth radionuclide.

The weighted radionuclide intake may be written

$$\sum_{k=1}^{r} w_k d_k \ ,$$

which may be rewritten

$$\sum_{k=1}^{r} w_k \sum_{j=1}^{n} c_{kj} x_j \ ,$$

or

$$\sum_{j=1}^{n} \sum_{k=1}^{r} w_k c_{kj} x_j \ .$$

We may now write the linear programming problem of minimizing

$$\sum_{j=1}^{n} \sum_{k=1}^{r} w_k c_{kj} x_j$$

subject to

$$x_j \geq 0 \ , \qquad j = 1 \ , \ 2 \ , \ \ldots \ , \ n$$

and

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \ , \qquad i = 1 \ , \ 2 \ , \ \ldots \ , \ m \ .$$

Maximizing Nutrition Subject to Radionuclide Bounds[3]

Let us define the bound

t_k = the maximum permissible amount of intake of the kth radionuclide .

We may write the constraint that the kth radionuclide must not exceed a given threshold

$$\sum_{j=1}^{n} c_{kj} x_j \leq t_k \ , \qquad k = 1 \ , \ 2 \ , \ \ldots \ , \ r \ .$$

If we call "nutrition" the weighted intake of the nutrients considered, defining the weight

u_i = the weight of the ith nutrient,

we may express the "nutrition" N of a diet with m nutrients by

$$N = \sum_{i=1}^{m} u_i \sum_{j=1}^{n} a_{ij} x_j \ .$$

Subject to the original nutrient constraints

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \ , \qquad i = 1 \ , \ 2 \ , \ \ldots \ , \ m \ ,$$

---

[3] This model was suggested by Dr. A. J. Goldman, Operations Research Section, Applied Mathematics Division, National Bureau of Standards.

and to

$$x_j \geq 0 \ , \qquad j = 1 \ , \ 2 \ , \ \ldots \ , \ n \ ,$$

$$\sum_{j=1}^{n} c_{kj} x_j \leq t_k \ , \qquad k = 1 \ , \ 2 \ , \ \ldots \ , \ r \ ,$$

the linear programming problem is to maximize

$$\sum_{i=1}^{m} u_i \sum_{j=1}^{n} a_{ij} x_j \ .$$

Considering the above inequalities it may be seen that it is possible to have no diets which satisfy the nutrient requirements and fall within the bounds. The nutrient requirements could be dropped or relaxed if radionuclide concentrations in foods were high enough to result in no solution to the programming model as given. The resultant solution if nutrient requirements were dropped would give the maximum "nutrition" possible.

Minimizing Quadratic Weighted Intake of Several Radionuclides

Instead of writing the weighted radionuclide intake as a linear sum

$$\sum_{k=1}^{r} w_k d_k \ ,$$

let us define a new symbol for the weighting factor

q_k = the weight assigned to the kth radionuclide,

and write the weighted radionuclide intake as a quadratic sum

$$\sum_{k=1}^{r} q_k d_k^{\,2} \ .$$

Minimization of this weighted radionuclide intake may be written as a quadratic programming problem, to minimize

$$\sum_{k=1}^{r} q_k \left(\sum_{j=1}^{n} c_{kj} x_j\right)^2$$

subject to

$$x_j \geq 0 \ , \qquad j = 1 , 2 , \ldots , n$$

and

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i \ , \qquad i = 1 , 2 , \ldots , m \ .$$

It is not suggested that a quadratic form is more appropriate than a linear form; the quadratic form is given to show a richer extension which has been computationally solved. Also, it will be very difficult even to assign linear weights to the radionuclides, as will be discussed later; hence, one would probably not use a quadratic form at this time in an implementation of the model.

FORMS OF SOLUTION

Solving for One Optimal Diet

The linear programming problems of minimizing intake of one radionuclide, of minimizing linearly weighted intake of several radionuclides, and of maximizing nutrition subject to radionuclide bounds could be solved by the simplex algorithm to yield an optimal diet. The constraints could be numerous and still be handled by available computer programs.

The quadratic programming problem of minimizing quadratic weighted intake of several radionuclides could be solved by Wolfe's modification of the simplex method (13). There is a computer program available which uses Wolfe's algorithm to arrive at an optimal solution (9).

Solving for the Set of All Optimal Diets

One optimal diet, however, would not be the most appropriate solution to the programming models presented. Sufficient quantities of foods selected in the optimal diet might not be available to feed the desired numbers of people; and also it is natural that people would like to choose their own diets, if possible, for variety and individual palatability.

All of the optimal solutions to the linear programming problem may be found by using the method proposed by Balinski (1). His method uses the basic pivotal operation of the revised simplex method. It has been programmed and is available in the SHARE library (8).

Solving for All Extreme Point Diets

Each of the linear inequality models presented above defines a convex polyhedral set, which has extreme point (or vertex) solutions. A very desirable feature of obtaining the set of all extreme point solutions for the cases where the linear inequalities define the nutrient constraints is that this set does not change with changes in the radioactivity of the foods. The objective function of most of the linear inequality models does change with changes in the radionuclide content of the foods, however; so for each value of the criterion function it could be evaluated at each of the previously determined extreme points. From the set of all extreme point diets could be selected the set of all optimal diets relative to the criterion function, and some near-optimal diets if desired. Convex combinations could be formed to generate more diets (with intermediate values of the criterion function) if this would be desirable.

It is very likely that the criterion function would be different for various geographic localities, since it depends upon the radionuclide content of foods. Also, if the nutrient requirements and the nutrient content of foods are fixed, the dietary constraints would be fixed over all time periods, while the criterion function would be changing.

The method of Balinski (1) mentioned above for finding all optimal solutions to linear programming problems also can be used to compute all extreme point solutions. The computer program for computing all optimal solutions (8) can also be used to compute all extreme point solutions.

The double description method developed by Motzkin, Raiffa, Thompson and Thrall (7) may also be used to find all extreme point solutions to a system of linear inequalities. This method generates a convex polyhedron by introducing linear inequalities one at a time and evaluating all of the vertices at each stage. John E. Bishop and the present writer have developed a computer program with limited capabilities to implement this procedure. Balinski's algorithm, however, can handle problems of very large size.

Interpretation of the Dual Variables

For each of the linear programming models which have been presented above there exists a dual problem, the solution of which may furnish valuable information. The present models could be more fully interpreted by considering the dual variables. For instance, in the model with fixed nutrient constraints and criterion function of minimizing weighted radionuclide intake, the dual variables would give insight into the intake associated with introducing or substituting foods not presently included in the optimal diet(s). An application of any of the models would profitably include a study of interpretation of the dual variables.

EXAMPLE

Input

An example using available data of radionuclide measurements for selected radionuclides and food groups, nutrient contents of selected representative foods within the food groups of the measurement data, and recommended dietary allowances of the nutrients included has been constructed.

Table 1 contains published data (11) of measured radionuclide contents of selected food groups in picocuries[4] per gram of food. Missing observations are denoted by an asterisk (*). For purposes of the example, the radionuclides in this sample are weighted equally ($w_k = 1, \quad k = 1, 2, \ldots, r$), and the missing observations are assumed to have

[4] Throughout this example the term picocuries (abbreviated pc) is used rather than the term micromicrocuries (μμc).

zero radionuclide content. This yields coefficients of the objective function

$$\sum_{k=1}^{r} w_k c_{kj} \quad , \qquad j = 1 , 2 , \ldots , n$$

equal to the sum of each column of table 1. The coefficients of the objective function are given in table 2.

Table 3 contains data on the content of a number of nutrients in foods which were purposively selected to be representative of the food groups of the data in table 1. The nutrient data in table 3 are from a standard food value reference (2). Table 4 contains recommended dietary allowances of the nutrients included in table 3 for a 25-year-old man, taken from the same food value reference.

Results

Using the double description method, vertices of the convex polyhedral set defined by the nutrient constraints were calculated. The 10 vertex diets with lowest weighted radionuclide intake are given in table 5.

It should be noted that the prevalence of "a selected pastry" in the vertex diets of table 5 can be explained by the relatively high values of some nutrients and the relatively low value of the coefficient of the objective function.

DISCUSSION

If data on radionuclide content of foods obtained by various measurement techniques are to be used for information or guidance relative to selection of diets, the models presented herein should prove to be useful. These models enable nutrient requirements to be satisfied while weighted radionuclide intake is reduced as much as possible.

The main limitations to use of the models are as follows:

1. Weighting factors must be known for the radionuclides considered if more than one radionuclide is included.

2. The solution is valid only so far as the measurements of radionuclide content of foods are correct. If these measurements are not certain but may be stated in terms of probability distributions, the problem becomes a stochastic programming problem and must be analyzed further.

3. Different types of people require different nutrient levels. This can be handled by separating the population into sex and age groups, but a separate solution must be obtained for each group.

Some of the advantages of the models are as follows:

1. There presently exists an extensive literature in mathematical programming which could be a rich source of interpretive ideas and types of analysis. Another highly developed area is that of available algorithms and computer programs to obtain solutions to various models.

2. The models provide a framework for integrating nutrient requirements and radiation measurement data.

3. The existence of models of this type which require weighting factors may furnish additional motivation for developing comparison criteria for biological damage from various radionuclides.

ACKNOWLEDGMENTS

Dr. A. J. Goldman contributed to the research described in this paper by several helpful discussions of the models, including suggestion of the model containing radionuclide bounds. Mr. David Rosenblatt contributed significantly to the presentation of results, including review of several drafts of the manuscript. Mr. Leroy L. Cook furnished administrative support and helpful comments.

TABLE 1.--Radionuclide Content of Food Groups (pc/gram)*

Radionuclides	Food Groups									
	Eggs	Fruits	Pastries	Cereals	Meats	Leafy Vegetables	Root Vegetables	Legumes and Corn	Rice	Milk
Sr^{89}	**	**	0.0	**	0.0	.0210	.0460	.0320	**	.0550
Sr^{90}	.0030	.0037	.0106	.0124	.0010	.0039	.0125	.0075	.0012	.0085
Ba^{140}	0.0	.0024	0.0	0.0	0.0	.0122	.0028	0.0	0.0	.0240
Ce^{137}	.0020	.0634	.0750	.1135	.1600	.0490	.0533	.0570	.0950	.0750
Zn^{65}	.0060	.0030	**	**	.0170	.0120	.0100	.0040	**	.0040
Zr^{95} and Nb^{95}	**	.0060	.0050	.0500	.0060	.0690	.0130	.0130	.0050	0.0
Ru^{106} and Rh^{106}	0.0	.0220	.0150	.1340	.0260	.2200	.0600	.0830	.0190	0.0

*Source: Reference (11)
**Denotes no data available

TABLE 2.--Coefficients of the Objective Function with Equally Weighted Radionuclides (weighted pc/gram)

	.0110	.1005	.1056	.3099	.2100	.3871	.1976	.1965	.1202	.1665

TABLE 3.--Nutrient Content of Specific Foods (nutrient units/gram)*

Nutrients and Units	Foods									
	Egg, Fried	Apple, Medium, Raw	A Selected Pastry	Prepared Dry Cereal**	Beef Hamburger, Cooked	Lettuce, Headed, Raw	Carrots, Raw	Corn, Yellow, Canned w/Liquid	Rice, White, Cooked	Milk, Whole, Fresh
Calories	2.200	.507	3.980	3.670	2.330	.140	.420	.664	1.200	.680
Proteins (gm)	.122	.003	.074	.148	.258	.012	.012	.020	.025	.035
Calcium (mg)	.540	.050	.543	1.590	.140	.220	.390	.039	.077	1.180
Iron (mg)	.026	.003	.017	.042	.038	.004	.008	.006	.003	.001
Vitamin A (IU) (Internat.Units)	14.040	.800	3.980	0.0	0.0	5.400	120.000	2.030	0.0	1.600
Thiamin (mcg) (microgram)	.800	.330	2.200	7.770	.760	.400	.600	.274	.119	.400
Riboflavin (mcg)	2.600	.270	2.260	1.410	2.040	.800	.600	.508	.060	1.700
Niacin (mg)	0.0	.001	.014	.019	.052	.002	.005	.009	.004	.001

*Source: Reference (2)
**"Cheerios" are used as example in this tabulation.

TABLE 4.--Recommended Dietary Allowances (nutrient units)*
(25-year-old man)

Calories	3200
Proteins	65 gm
Calcium	800 mg
Iron	12 mg
Vitamin A	5000 IU
Thiamin	1600 mcg
Riboflavin	1600 mcg
Niacin	16 mg

*Source: Reference (2)

TABLE 5.--Vertex Diets (in grams) with Lowest Weighted Radionuclide Intake (weighted pc)

Foods										Weighted Radionuclide Intake
Egg, Fried	Apple, Medium, Raw	A Selected Pastry	Prepared Dry Cereal*	Beef, Hamburger, Cooked	Lettuce, Headed, Raw	Carrots, Raw	Corn, Yellow, Canned, w/Liquid	Rice, White, Cooked	Milk, Whole, Fresh	
1707.7				307.7						83.4
1217.8		196.8		253.6						87.4
1267.9			46.8	291.8						89.7
1012.3			69.6	282.5					87.5	106.6
897.5			176.5	248.0						116.6
356.4		1118.9								122.1
227.5		534.9		154.7					309.4	143.0
24.2		1107.9							157.1	143.4
1035.9			48.7	260.0		324.9				145.3
		1106.2				2.7			168.0	145.3

* "Cheerios" are used as example in this tabulation.

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